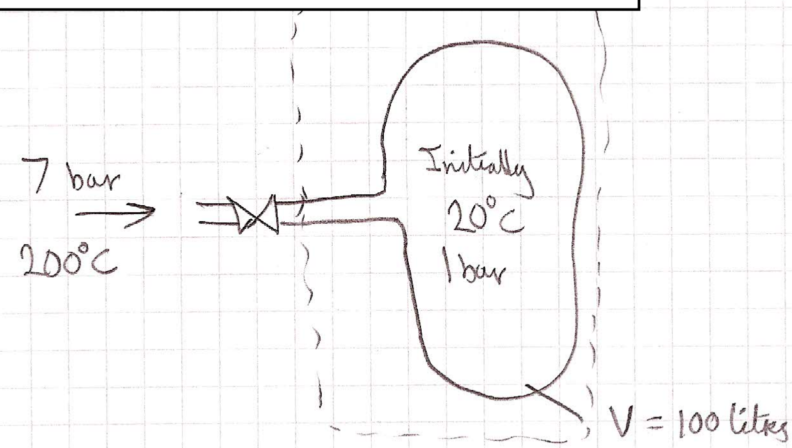
Q1



Initial condition in vessel $T_1 = 20°$ $p_1 = 1$ bar

Final condition in vessel $T_2 = ?$ $p_2 = 7$ bar

NSFEE becomes:

$$0 = -\dot{m}_i h_i + \frac{d}{dt}(m_{cv} u)$$

Air enters at constant condition so h_i = constant

Integrating from start to finish gives

$$0 = -h_i \int_1^2 dm_i + m_{cv_2} u_2 - m_{cv_1} u_1$$

$$= -h_i (m_{cv_2} - m_{cv_1}) + (m_{cv_2} u_2 - m_{cv_1} u_1)$$

Let $h_i = c_p T_i$ Temperatures must be in Kelvin

$u = c_v T$.

$$m_{cv} = \frac{pV}{RT}$$

$$c_p T_i \frac{\cancel{V}}{\cancel{R}} \left(\frac{p_2}{T_2} - \frac{p_1}{T_1} \right) = c_v \frac{\cancel{V}}{\cancel{R}} \left(\frac{p_2}{\cancel{T_2}} \cancel{T_2} - \frac{p_1}{\cancel{T_1}} \cancel{T_1} \right)$$

$$\frac{c_p}{c_v} = \gamma = 1.4 \text{ for air}$$

Re arrange to give

$$T_2 = \frac{T_1}{\frac{p_1}{p_2}\left[\frac{T_1}{T_i \gamma} \left(\frac{p_2}{p_1} - 1 \right) + 1 \right]}$$

$$T_2 = \frac{293}{\frac{1}{7}\left[\frac{1}{1.4}(7-1)+1\right]} = \underline{388\ K} \quad (115^\circ C)$$

Initial mass of air in tank $\quad m_{cv} = \dfrac{p_1 V}{R T_1} = \dfrac{10^5 \times 0.1}{287.1 \times 293}$

$$= 0.1189\ kg$$

Final mass of air in tank $= \dfrac{p_2 V}{R T_2} = \dfrac{7 \times 10^5 \times 0.1}{287.1 \times 388}$

$$= 0.6284\ kg$$

So mass of air entering tank is $\underline{0.51\ kg}$

When air in tank cools to 20°C pressure is given by

$$p = \frac{mRT}{V} = \frac{0.6284 \times 287.1 \times 293}{0.1} = \underline{5.29\ bar}$$

Exergy of air entering tank is <u>flow exergy</u>

$$E_1 = m\left((h_i - h_o) - T_o(s_i - s_o)\right)$$

$h_i - h_o = 0$ as air enters at T_o (293 K), $T_i = T_o$

$$s_i - s_o = c_p \log_e\left(\frac{T_i}{T_o}\right) - R \log_e\left(\frac{p_i}{p_o}\right)$$

$$= 0 - 287.1 \log_e(7)$$

$$= -558.6\ J/kg\ K$$

$$\therefore\ \underline{E_1} = 0.51\left(-293(-558.6)\right) = \underline{83.5\ kJ}$$

Exergy of air in tank immediately after filling

Exergy of a Closed System $\Xi_2 = m_{cv_2}\left(u_2 - u_0\right) + p_0\left(v_2 - v_0\right) - T_0\left(s_2 - s_0\right)$

v_2 = specific vol of air in tank $= \frac{RT_2}{p_2} = \frac{287.1 \times 388}{7 \times 10^5} = 0.159 \ m^3/kg$

v_0 = specific volume of air at env temp & pressure $= \frac{RT_0}{p_0} = \frac{287.1 \times 293}{10^5} = 0.841 \ m^3/kg$

$$s_2 - s_0 = c_p \log_e\left(\frac{T_2}{T_0}\right) - R \log_e\left(\frac{p_2}{p_0}\right)$$

$$= 1005 \log_e\left(\frac{388}{293}\right) - 287.1 \log_e\left(\frac{7}{1}\right)$$

$$= 282.2 - 558.7$$

$$s_2 - s_0 = -276.47$$

$\left(c_v = \frac{c_p}{\gamma}\right)$

$$\Xi_2 = 0.6284\left[\frac{1005}{1.4}\left(388-293\right) + 10^5\left(0.159-0.841\right) - 293\left(-276.47\right)\right]$$

$$\underline{\underline{\Xi_2 = 50.9 \ kJ}}$$

When air in tank cools to 20°C, specific volume remains same as mass in tank does not change.

$$\text{So} \quad \Xi_3 = 0.6284\left[0 + 10^5\left(0.159-0.841\right) - 293\left(0 - 287.1 \log_e\left(\frac{5.29}{1}\right)\right)\right]$$

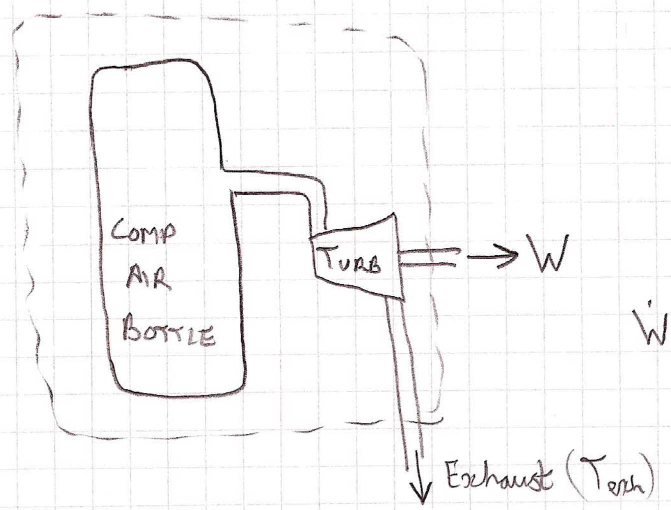
$$\underline{\underline{\Xi_3 = 45.2 \ kJ}}$$

There is an initial loss of exergy as air enters tank, as it throttles on entering (initially air in tank is at 1 bar, but air enters from supply at 7 bar). Loss of exergy is 83.5 − 50.9 = 32.6 kJ (39% of initial exergy in air.)

On cooling air loses a further $(50.9 - 45.2) = 5.7$ kJ
6.8% of initial exergy.

In total the air finally in tank at equilibrium only has 54% of the initial exergy.
Storing energy as compressed air in this way is not very efficient.

Q2



$\dot{W} = 4\,kW$ for 30 secs
(average power)

In vessel initially $p_1 = 34$ bar $T_1 = 15°C$ (288K)

In vessel finally $p_2 = 3.4$ bar

$T_{exh} = -150°C$ (123K)

Air in bottle expands isentropically & adiabatically so pv^γ

This means $T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\gamma = 1.4$ for air.

$$\therefore T_2 = 288 \left(\frac{3.4}{34}\right)^{\frac{0.4}{1.4}} = 149.2\,K \quad (-124°C)$$

Apply NSFEE

$$-\dot{W} = h_{exh}\dot{m}_{exh} + \frac{d}{dt}(mU)_{bottle}$$

(−ve as work is being done by the system)

Integrating gives

$$-\int \dot{W}\,dt = h_{exh}\int_1^2 dm_{exh} + (mU)_2 - (mU)_1 \quad -(1)$$

$$h_{exh}\int_1^2 dm_{exh} = -h_{exh}\int_1^2 dm)_{bottle}$$

Mass in bottle goes down as mass leaves system
h_{exh} is constant

$$-\int \dot{W}\,dt = \text{average power output} \times \text{time}$$

$$= -4000 \times 30 = -120 \text{ kJ}$$

$$-h_{exit} \int_1^2 d(m)_{bottle} = -c_p T_e (m_2 - m_1)$$

$(\Delta h = c_p \Delta T)$

[taking a datum of $h = 0$ at 0 K gives $h = c_p T$]

$$(mu)_2 - (mu)_1 = m_2 c_v T_2 - m_1 c_v T_1$$

$$m = \frac{pV}{RT} \qquad m_1 = \frac{34 \times 10^5}{287.1 \times 288} V = 41.12V$$

$$m_2 = \frac{3.4 \times 10^5}{287.1 \times 149.2} V = 7.94V$$

Substituting into (1) gives

[Use $c_p = 1.005$ kJ/kgK
$c_v = \frac{c_p}{\gamma} = 0.718$ kJ/kgK]

$$-120 = -1.005 \times 123 \,(7.94V - 41.12V)$$
$$+ 7.94V \times 0.718 \times 149.2 - 41.12V \times 0.718 \times 288$$

$$V = 0.0338 \text{ m}^3$$

Volume of bottle is 33.8 litres